Factors

This problem gives you the chance to:
- work with factors of numbers up to 30

A factor of a number divides into the number exactly.

This table shows all the factors of most of the numbers up to 30.

Number	Factors	Number of factors
1	1	1
2	1, 2	2
3	1, 3	2
4	1, 2, 4	3
5	1, 5	2
6	1, 2, 3, 6	4
7	1, 7	2
8	1, 2, 4, 8	4
9	1, 3, 9	3
10	1, 2, 5, 10	4
11	1, 11	2
12	1, 2, 3, 4, 6, 12	6
13	1, 13	2
14	1, 2, 7, 14	4
15	1, 3, 5, 15	4

Number	Factors	Number of factors
16	1, 2, 4, 8, 16	5
17	1, 17	2
18	1, 2, 3, 6, 9, 18	6
19	1, 19	2
20	1, 2, 4, 5, 10, 20	6
21	1, 3, 7, 21	4
22	1, 2, 11, 22	4
23	1, 23	2
24	1, 2, 3, 4, 6, 8, 12, 24	8
25	1, 5, 25	3
26	1, 2, 13, 26	4
27	___ ___ ___ ___	4
28	___ ___ ___ ___ ___ ___	6
29	___ ___	2
30	1, 2, 3, 5, 6, 10, 15, 30	8

1. Write the factors of the numbers 27, 28, and 29 in the table.

2. The numbers 1 and 4 have an odd number of factors.

   a. Write down all the numbers up to 30 that have an odd number of factors.

   1, 4 , _____, _____, _____

   b. Complete this sentence:

   All the _____ *numbers up to 30 have an odd number of factors.*

3. The number 10 has two odd factors (1 and 5).
   It also has two even factors (2 and 10).

   The number 18 has three odd factors (1, 3 and 9).
   It also has three even factors (2, 6 and 10).

   a. Write down all the numbers up to 30 that have an equal number of odd and even factors.

   2, 6, 10, _____, 18, _____, _____, _____

   b. Describe two patterns you can see in the above sequence of numbers.

_____

_____

_____

_____

Task 3: Factors		Rubric	
The core elements of performance required by this task are: • work with factors of numbers up to 30 Based on these, credit for specific aspects of performance should be assigned as follows		points	section points
1	Gives correct answers: 27: **1, 3, 9, 27** 28: **1, 2, 4, 7, 14, 28** 29: **1, 29**	 1 1 1	 3
2 a	Gives three correct answers with no extras: **9, 16, 25** **Partial credit** Gives two correct answers.	 2 (1)	
b	Gives correct answer: **square**	1	3
3.a	Gives correct answers: **14, 22, 26, 30**	 1	
b	Describes two correct patterns such as: They are all even numbers. The sequence increases in fours. They are all twice an odd number	 2 x 1	 3
	Total Points		**9**

Factors

Work the task. Look at the rubric. What are the big mathematical ideas being assessed in this task?

Most students knew how to make the factors for part 1. Find the work of students who missed this part and consider their work separately. What would be the next steps for these students? How could you assess their basic understanding of multiplication and division?

While many students could continue the pattern for part 2 students had a difficult time describing the pattern. How many of your students put:

Square numbers	No response	Composite numbers	Prime	Even	Factors	Odd	Other

How can we pose meaningful discussions that give students a reason to use academic language in their explanations? What types of activities help students develop academic language? What misconceptions might have led to the use of some of these incorrect terms?

Look at the work of students in part 3. How many of your students put:

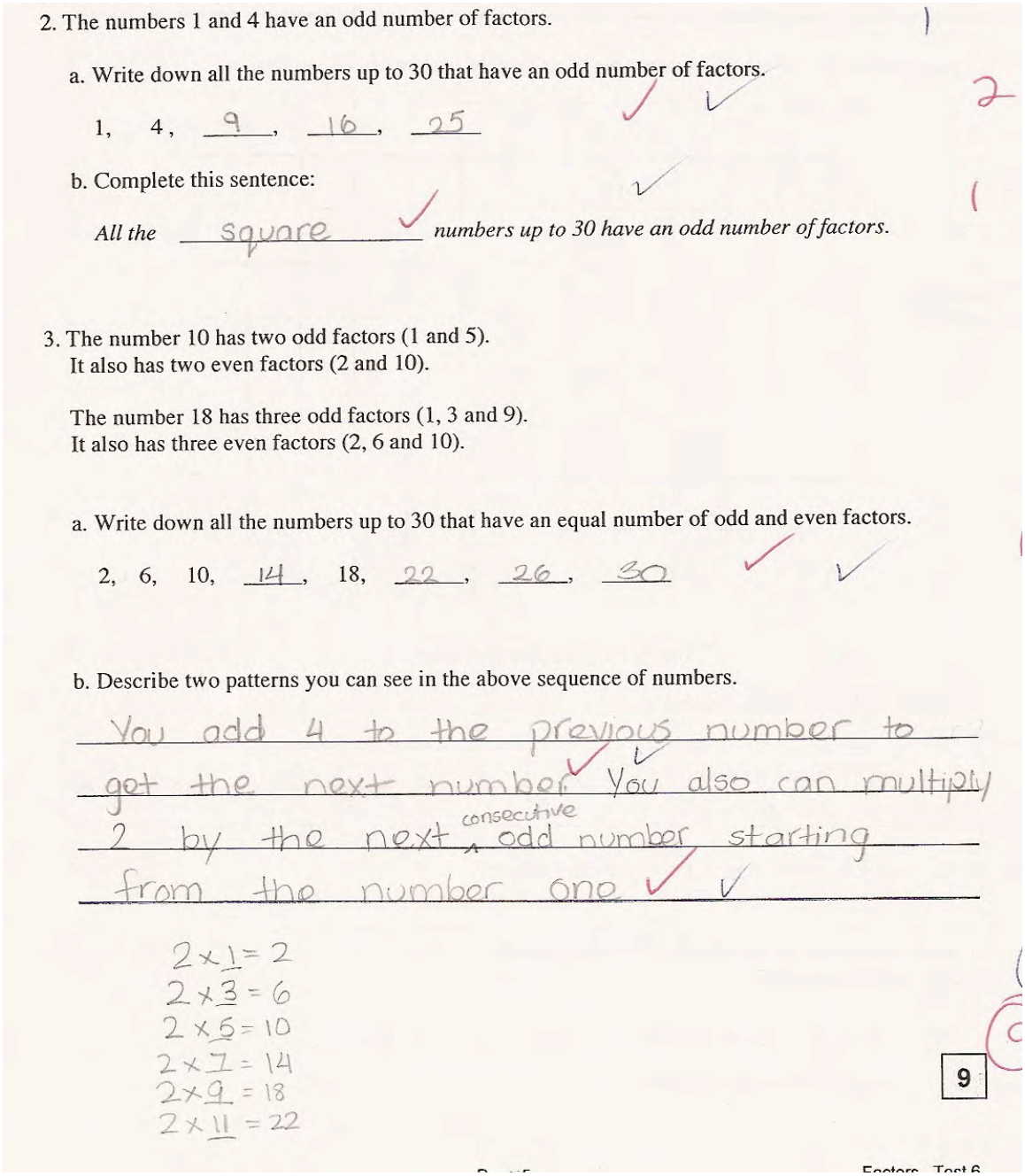
Even #'s	Goes up by 4's	2 times an odd #	Only one clue	No response	Other

What are the attributes of numbers that your students seem comfortable with or that are easily noticed? What are attributes that are less obvious to students? What opportunities have students had to sort numbers by attributes?

Looking at Student Work on Factors

Student A has a clear grasp of the properties of numbers and the associated vocabulary. Notice the documentation to back up the second pattern in 3b.

Student A



2. The numbers 1 and 4 have an odd number of factors.

a. Write down all the numbers up to 30 that have an odd number of factors.

1, 4, 9, 16, 25

b. Complete this sentence:

All the square *numbers up to 30 have an odd number of factors.*

3. The number 10 has two odd factors (1 and 5).
It also has two even factors (2 and 10).

The number 18 has three odd factors (1, 3 and 9).
It also has three even factors (2, 6 and 10).

a. Write down all the numbers up to 30 that have an equal number of odd and even factors.

2, 6, 10, 14, 18, 22, 26, 30

b. Describe two patterns you can see in the above sequence of numbers.

You add 4 to the previous number to get the next number. You also can multiply 2 by the next consecutive odd number starting from the number one

$2 \times 1 = 2$
$2 \times 3 = 6$
$2 \times 5 = 10$
$2 \times 7 = 14$
$2 \times 9 = 18$
$2 \times 11 = 22$

9

Student B also has a nice strategy for thinking about the factors in part 1 (numerical values are given on the chart). Notice the drawings to confirm the numbers are square in part 2.

Student B

1. Write the factors of the numbers 27, 28, and 29 in the table.
I know that all # have one and itself as factors. then I know that 3 & 9 are factors of 27. 7x4=28 2x14=28 29=prime means only 2 factors

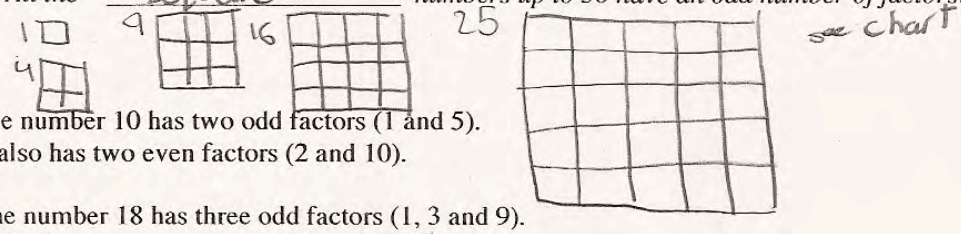
2. The numbers 1 and 4 have an odd number of factors.

a. Write down all the numbers up to 30 that have an odd number of factors.

1, 4, 9, 16, 25 ✓ ✓ 2

b. Complete this sentence:

All the square *numbers up to 30 have an odd number of factors.* ✓ ✓



see chart

3. The number 10 has two odd factors (1 and 5).
It also has two even factors (2 and 10).

The number 18 has three odd factors (1, 3 and 9).
It also has three even factors (2, 6 and 18).

a. Write down all the numbers up to 30 that have an equal number of odd and even factors.

2, 6, 10, 14, 18, 22, 26, 30 ✓ ✓
see Chart

b. Describe two patterns you can see in the above sequence of numbers.

I see that all the numbers are even ✓ ✓ and the difference between the numbers is four ✓ so the pattern is 4 4

How do students learn mathematical vocabulary? Student C understands the properties of the numbers in part 2, but has not yet mastered the academic vocabulary. Student D either doesn't see the commonality in part 2 or doesn't know the vocabulary word. In part 3b the student sees the same pattern as Student A, multiplying by odd numbers, but doesn't have the formal language for expressing the pattern. *What activities do you do regularly in your classroom to give students the opportunity to purposefully practice using mathematical language? Do you provide enough tasks that help students notice more complex patterns involving a variety of number properties?*

Student C

2. The numbers 1 and 4 have an odd number of factors.

a. Write down all the numbers up to 30 that have an odd number of factors.

1, 4, 9, 16, 25

b. Complete this sentence:

All the numbers that have a factor that can be multiplied against itself to equal the number *numbers up to 30 have an odd number of factors.*

Student D

2. The numbers 1 and 4 have an odd number of factors.

a. Write down all the numbers up to 30 that have an odd number of factors.

1, 4, 9, 16, 25

b. Complete this sentence:

All the _____ *numbers up to 30 have an odd number of factors.*

3. The number 10 has two odd factors (1 and 5).
It also has two even factors (2 and 10).

The number 18 has three odd factors (1, 3 and 9).
It also has three even factors (2, 6 and 10).

a. Write down all the numbers up to 30 that have an equal number of odd and even factors.

2, 6, 10, 14, 18, 22, 26, 30

b. Describe two patterns you can see in the above sequence of numbers.

- They increase by 4
- If you add 1 to each number it equals the next number. The next number you do this to, it skips one and equals the next.

skips 1, skips 2, skips 3

2 6 10 14 18 22 26 30 ? 34

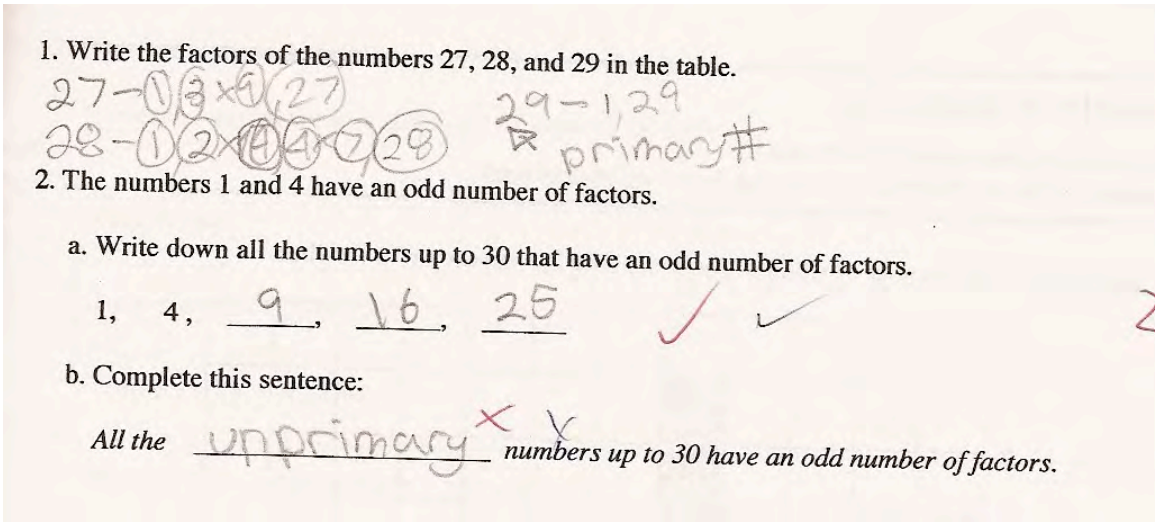
2 + 1 = 3 · 2 = 6 / 6 + 1 = 7 · 2 = 14 / 10 + 1 = 11 · 2 = 22 /

18 + 1 = 19 · 2 = 38

9

Student E also struggles with vocabulary. The student knows something about prime numbers and coins his own word for composite numbers.

Student E



1. Write the factors of the numbers 27, 28, and 29 in the table.

27 – 1, 3, 9, 27
28 – 1, 2, 4, 7, 28
29 – 1, 29 ☆ primary #

2. The numbers 1 and 4 have an odd number of factors.

a. Write down all the numbers up to 30 that have an odd number of factors.

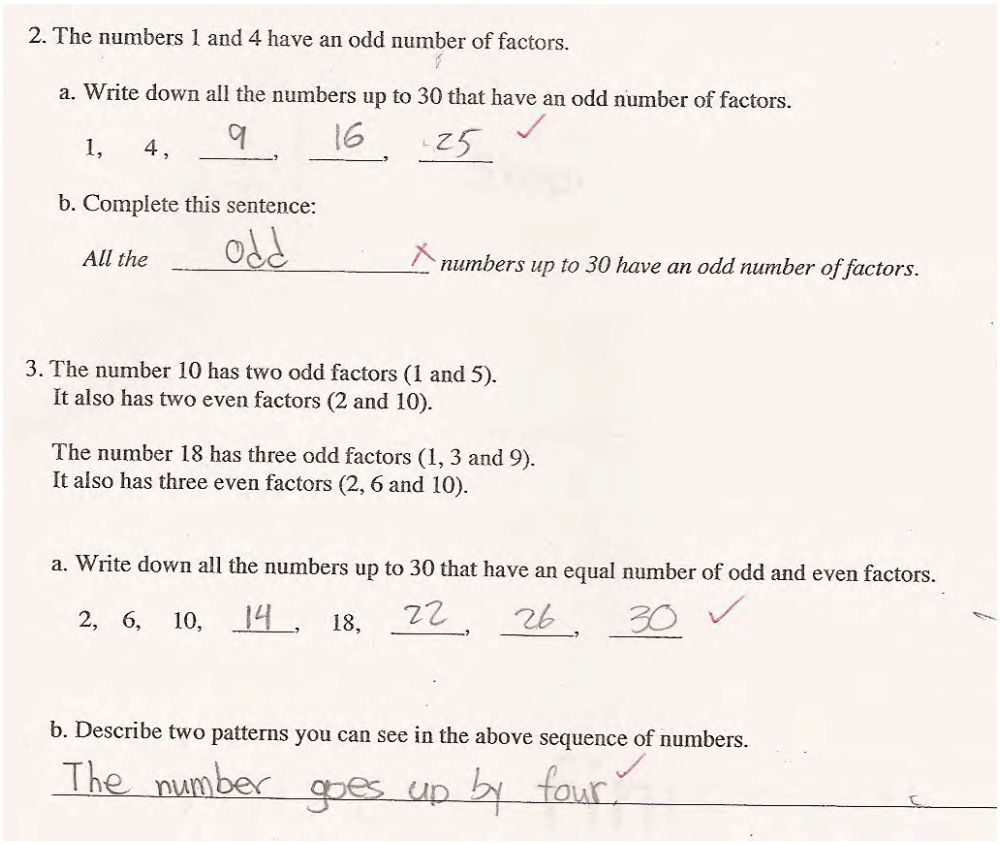
1, 4, 9, 16, 25 ✓ ✓

b. Complete this sentence:

All the unprimary ✗ ✓ *numbers up to 30 have an odd number of factors.*

Some students noticed irrelevant properties. They don't have the habit of mind to check their conjectures against all the evidence. Many students, like Student F, total score on task = 7, did not persevere in looking for further patterns in part 3 once the first pattern was identified. *How do you help students develop productive habits of mind?*

Student F



2. The numbers 1 and 4 have an odd number of factors.

a. Write down all the numbers up to 30 that have an odd number of factors.

1, 4, 9, 16, 25 ✓

b. Complete this sentence:

All the odd ✗ *numbers up to 30 have an odd number of factors.*

3. The number 10 has two odd factors (1 and 5).
It also has two even factors (2 and 10).

The number 18 has three odd factors (1, 3 and 9).
It also has three even factors (2, 6 and 10).

a. Write down all the numbers up to 30 that have an equal number of odd and even factors.

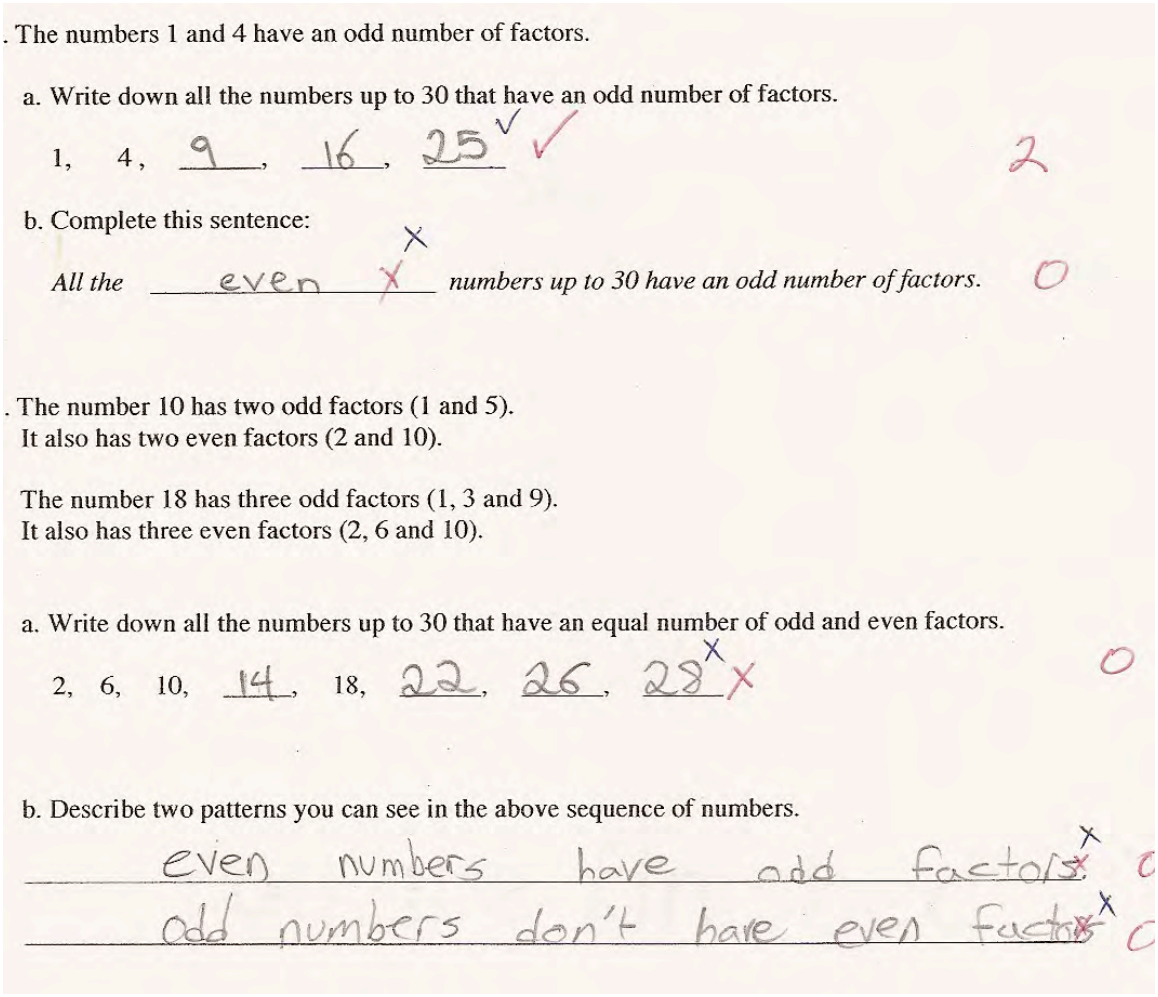
2, 6, 10, 14, 18, 22, 26, 30 ✓

b. Describe two patterns you can see in the above sequence of numbers.

The number goes up by four. ✓

Student G does not check her conjecture in part 2 against all the evidence. In part 3 the student gives patterns that don't seem to relate directly to the set of numbers listed in 3a.

Student G



. The numbers 1 and 4 have an odd number of factors.

a. Write down all the numbers up to 30 that have an odd number of factors.

1, 4, 9, 16, 25 ✓ ✓ 2

b. Complete this sentence:

All the even X *numbers up to 30 have an odd number of factors.* 0

. The number 10 has two odd factors (1 and 5).
It also has two even factors (2 and 10).

The number 18 has three odd factors (1, 3 and 9).
It also has three even factors (2, 6 and 10).

a. Write down all the numbers up to 30 that have an equal number of odd and even factors.

2, 6, 10, 14, 18, 22, 26, 28 X X 0

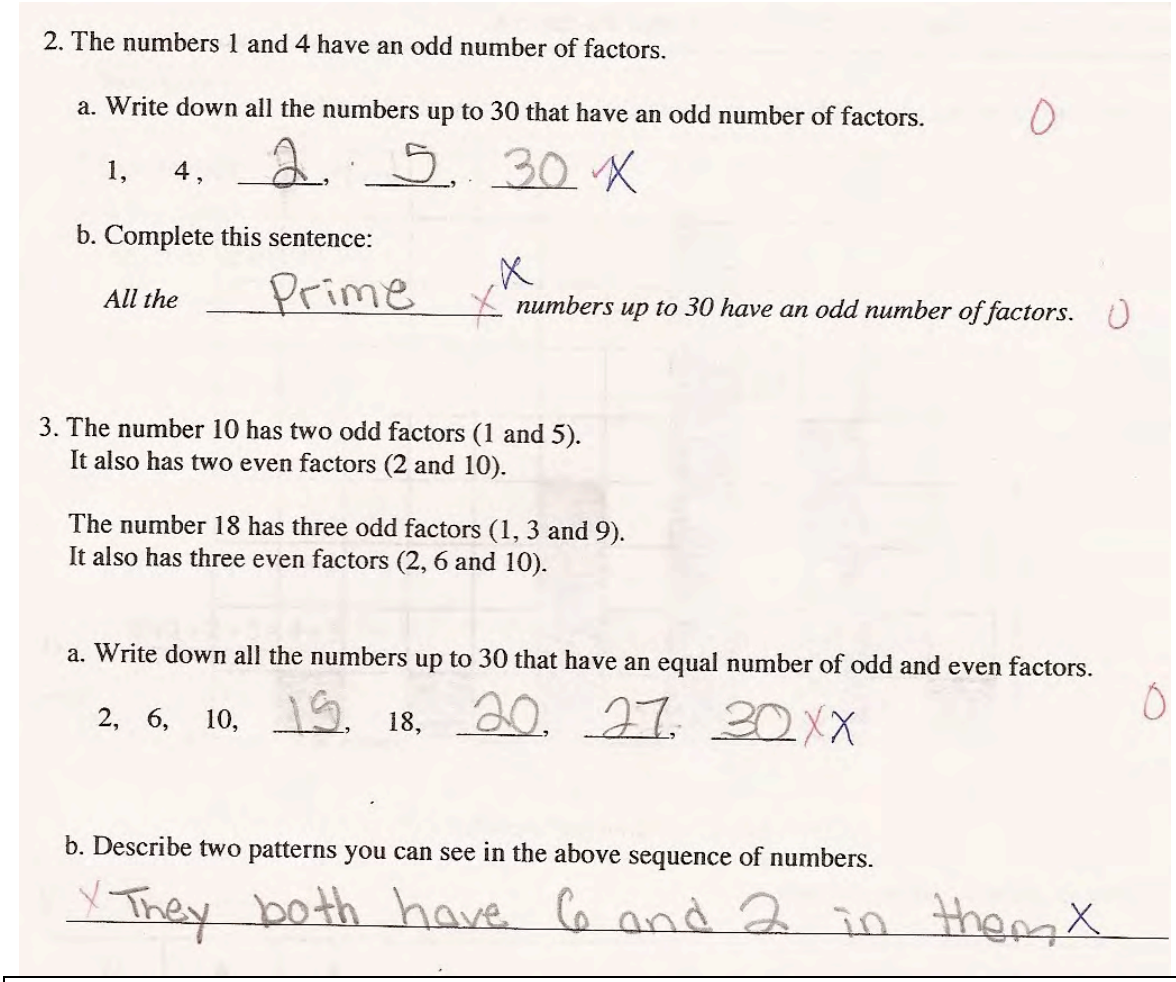
b. Describe two patterns you can see in the above sequence of numbers.

even numbers have odd factors X 0

odd numbers don't have even factors X 0

Student H is able to make a list of correct factors for all the numbers on the first page of the task. The student doesn't decipher the vocabulary in part 2a or 3a and therefore can't complete the lists. Notice that prime does not describe the choice of numbers in 2a and the description in 3b is unclear.

Student H



2. The numbers 1 and 4 have an odd number of factors.

a. Write down all the numbers up to 30 that have an odd number of factors. 0

1, 4, 2, 5, 30 X

b. Complete this sentence:

All the Prime X X *numbers up to 30 have an odd number of factors.* 0

3. The number 10 has two odd factors (1 and 5).
It also has two even factors (2 and 10).

The number 18 has three odd factors (1, 3 and 9).
It also has three even factors (2, 6 and 10).

a. Write down all the numbers up to 30 that have an equal number of odd and even factors.

2, 6, 10, 15, 18, 20, 27, 30 X X 0

b. Describe two patterns you can see in the above sequence of numbers.

X They both have 6 and 2 in them X

Some students could not make the list of factors for page one of the task. Student I could not interpret the instructions in part 2. Notice that the filled in word does not describe the numbers in Student I's list.

Student I

25	1, 5, 25	3
26	1, 2, 13, 26	4
27	1 2 4 6	4
28	1 2 3 8 10 12	6
29	1 2	2
30	1, 2, 3, 5, 6, 10, 15, 30	8

2. The numbers 1 and 4 have an odd number of factors. yes

a. Write down all the numbers up to 30 that have an odd number of factors.

1, 4, 3, 6, 9

b. Complete this sentence:

All the even *numbers up to 30 have an odd number of factors.*

3. The number 10 has two odd factors (1 and 5).
It also has two even factors (2 and 10).

The number 18 has three odd factors (1, 3 and 9).
It also has three even factors (2, 6 and 10).

a. Write down all the numbers up to 30 that have an equal number of odd and even factors.

2, 6, 10, 14, 18, 22, 26, 30

b. Describe two patterns you can see in the above sequence of numbers.

the pattern is 4 because in each space there is a four.

6th Grade Task 3 Factors

Student Task	Work with factors of numbers up to 30. Recognize and identify patterns based on number properties or number attributes.
Core Idea 1 Number and Operation	**Understand number systems, the meanings of operations and ways of represent numbers, relationships, and number systems.** • Use factors, multiples, prime factorization and relatively prime numbers to solve problems.

Based on teacher observations, this is what sixth graders know and are able to do:

- Make a list of factors for a given number
- Use a table to write down numbers with an equal number of odd and even factors
- Find at least one clue to fit the numbers with an equal number of odd and even factors
- Make a list of numbers with an odd number of factors

Areas of difficulty for sixth graders:

- Mathematical vocabulary
- Understanding properties of numbers, like square numbers or consecutive odd numbers
- Finding more complex patterns like factors of 2 and an odd number
- Looking for more than one pattern
- Not checking answers against all the available evidence

MARS Test Task 3 Frequency Distribution and Bar Graph, Grade 6

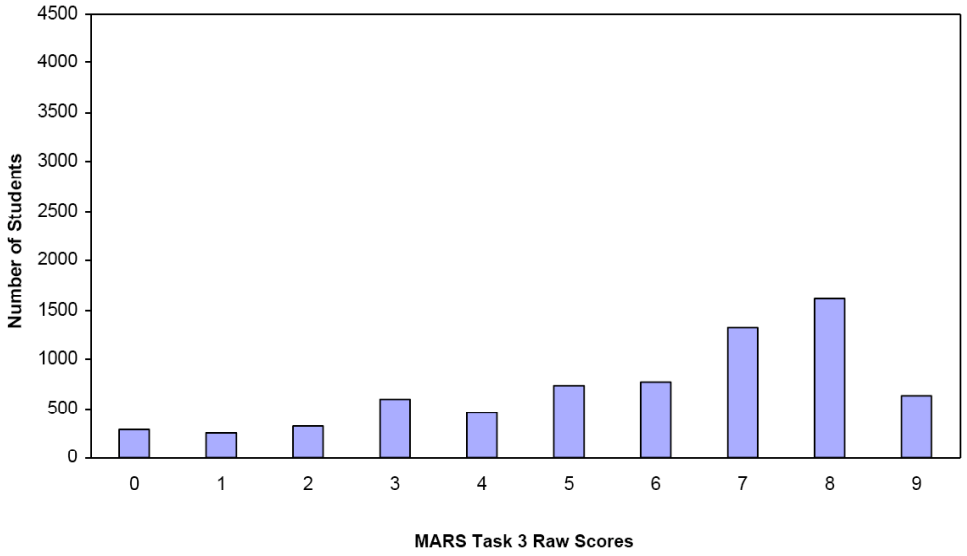
Task 3 - Factors

Mean: 5.83 StdDev: 2.47

Table 32: Frequency Distribution of MARS Test Task 3, Grade 6

Task 3 Scores	Student Count	% at or below	% at or above
0	283	4.1%	100.0%
1	247	7.6%	95.9%
2	316	12.2%	92.4%
3	591	20.7%	87.8%
4	460	27.3%	79.3%
5	724	37.7%	72.7%
6	759	48.6%	62.3%
7	1324	67.7%	51.4%
8	1617	91.0%	32.3%
9	628	100.0%	9.0%

Figure 41: Bar Graph of MARS Test Task 3 Raw Scores, Grade 6



The maximum score available for this task is 9 points.
The minimum score for a level 3 response, meeting standards, is 5 points.

Most students, 88%, could make a list of factors for the 3 numbers on page one of the task. Many students, 73%, could list the factors for the 3 numbers, and make a list of factors that had an odd number of factors. More than half the students, 51%, could also make a list of numbers with an equal amount of odd and even factors and find at least one pattern in those numbers. 32% of the students could find two patterns in the numbers with an equal number of odd and even factors. 9% of the students could meet all the demands of the task, including identifying square numbers. 4% of the students scored no points on this task. 2/3 of the students with this score did not attempt the task.

Factors

Points	Understandings	Misunderstandings
0	Only 1/3 of the students with this score attempted the task.	Students did not know how to find factors of numbers. A common error was to omit the number itself from the list of factors or give only some of the factors.
3	Students could fill in all the factors for the 27, 28, and 29.	Students had difficulty listing the numbers with an odd number of factors. Errors did not fit any pattern.
5	Students could find the factors for numbers and make a list of the numbers with an odd number of factors.	Students could not make a list of numbers with an equal number of odd and even factors.
7	Students could now make a list of numbers with an equal number of odd and even factors and find one pattern in the numbers.	25% of the students only listed one rule for part 3b.
8	Students could now write 2 rules for part 3b.	Students had difficulty describing the square numbers. 15% described them as even numbers. 13% left 2b blank. 13% put odd numbers. 7% said the numbers were prime. 7% filled in the blank with factors.
9	Students could list factors for numbers, sort them by factor properties, identify square numbers, and find two patterns for numbers with equal amounts of odd and even factors.	

Implications for Instruction

Students need to be able to find the factors of numbers. Students should also be able to look at a table of information and decide if a number has an odd or even number of factors. Students need to be able to look at a list of numbers and decide if it has the same amount of odd and even numbers.

Students at this grade level need to be transitioning from look at numbers individually to thinking about categories of numbers or systems of numbers. They should have opportunities to investigate numbers and their properties and look for commonalities and patterns.

Ideas for Action Research – Investigations with Number

Students at this grade level should begin to develop strategies for conducting investigations. They need rich tasks that push their skills at organizing information, perseverance, and thinking about cases and types of numbers. Students need opportunities in class to build convincing arguments to back up or defend their conjectures.

Here are two investigations on numbers from Fostering Algebraic Thinking by Mark Driscoll:

Something Nu

Consider the operation of counting the factors of a whole number. This function is usually called "ν" (the lower case Greek letter for "nu"). For example, the number 6 ha the factors 1,2,3, and 6, so $\nu(6) = 4$. Here's some practice:

1. If the input to ν is 5, what is the output? What if the input is 12?

2. What is $\nu(24)$? $\nu(288)$? $\nu(23 \times 3^2 \times 5^4)$?

3. Find some numbers that ν takes to 6.

4. Classify all numbers n so that $\nu(n) = 3$. Classify all numbers n so that $\nu(n)=2$.

5. What can you say about a number m if ν(nM0 = 12?

Differences of Squares

Which numbers can be expressed as the difference of two perfect squares?

How do these problems build perseverance? What properties of numbers to students investigate? How do students organize their information? What other problems can you give students to push the idea of experimenting with number properties and looking for patterns and generalizations?